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LAMINAR CONVECTION TO ROTATING CONES AND DISKS IN NON-NEWTONIAN POWER-LAW FLUIDS

R. N. SMITH* and R. GREIF
University of California, Department of Mechanical Engineering, Berkeley, CA 94720, U.S.A.

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Abstract—The mass transfer has been obtained for laminar flow about rotating cones and disks in non-Newtonian power-law fluids. An exact result based on a two term representation of the velocity field is presented. A comparison with previous analyses based on a one term velocity profile defines the range of validity of the earlier results.

NOMENCLATURE

a, b, dimensionless parameters appearing in equations (4) and (5);
c, concentration;
D, molecular diffusivity;
j_w, local wall mass flux;
J_w, average wall mass flux;
K, power law consistency index;
L, cone slant length;
n, power law index;
u, velocity component parallel to cone surface;
w, velocity component normal to cone surface;
x, coordinate parallel to cone surface;
z, coordinate normal to cone surface;
α, cone half-angle;
β_1, $\left(\dfrac{\partial u}{\partial z}\right)_{z=0}$;
β_2, $-\left(\dfrac{\partial^2 u}{\partial z^2}\right)_{z=0}$;
η, dimensionless transformation variable;
θ, dimensionless concentration $\left(\dfrac{c-c_\infty}{c_w-c_\infty}\right)$;
ρ, density;
ω, rotational speed;
A, B, N, N_1, functions of n defined in paper;
Re_L, Reynolds number;
Sc, Schmidt number;
$\overline{Sh}$, average Nusselt number.

Subscripts

w, wall;
∞, free stream.

INTRODUCTION

THE HEAT or mass transfer in a non-Newtonian fluid of the power-law class to rotating disks and cones in laminar flow has been treated experimentally and analytically by several investigators. Hansford and Litt [1] reported on the mass transfer from a rotating disk made of the solute diffusing to non-Newtonian solutions. They measured the rates of dissolution of benzoic acid and napthol into an aqueous solution of carboxymethylcellulose, and of benzoic acid into aqueous polyethylene oxide. The value of the diffusivity was then determined by comparing their experimental results with an approximate theoretical expression for the mass flux to a rotating disk at high Schmidt numbers. Later, Greif, Cornet and Kappesser [2] applied this theoretical expression to the experimental results on a rotating disk system to determine the molecular diffusivity of dissolved oxygen in an aqueous sodium chloride solution which was rendered non-Newtonian by adding various concentrations of Polyox WSR 301 (Union Carbide), a completely water soluble polymer of ethylene oxide.

Greif and Paterson [3] then obtained a more accurate result for the mass flux at high Schmidt numbers. This was obtained from an exact solution to the boundary layer equation and was based on a linear velocity distribution as suggested by Lighthill [4]. The result was used in conjunction with the experimental results of [2] to obtain improved values for the oxygen diffusivity. A similar theoretical expression valid for the rotating cone geometry was then used by Paterson *et al.* [5] with these diffusivity results. They compared the experimentally determined mass flux of dissolved oxygen to a 60° cone rotating in Polyox solution with the theoretical expression and obtained good agreement with the data. It is the purpose of the present analysis to extend the analysis of [3] and [5] to obtain a more accurate expression for the mass flux which should be applicable over a wider range of Schmidt numbers. This is accomplished by utilizing a two-term representation of the velocity field in the boundary layer as reported by Chao [6] to determine the heat or mass transfer in laminar forced convection for flow past a two-dimensional stationary body. The analysis of [6] has also been extended by Chao and Greif [7] for flow over a rotating body of revolution in a Newtonian fluid.

ANALYSIS

The geometry and coordinate system, including velocity components, for the flow near a rotating cone are shown in Fig. 1. In this coordinate system the mass transport, in the boundary layer is governed by

$$u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z} = D\frac{\partial^2 \theta}{\partial z^2} \tag{1}$$

*Present address, Rice University, Houston, TX 77001, U.S.A.

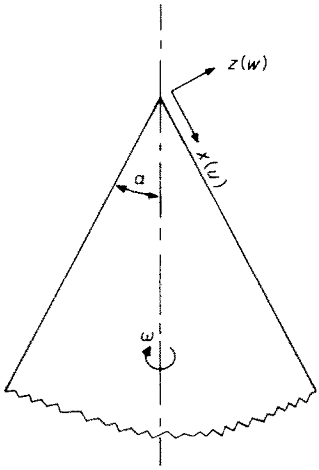

FIG. 1. Coordinate system.

subject to the following conditions: $\theta(x, 0) = 1$, $\theta(x, \infty) = 0$ and $\theta(0, \infty) = 0$. The two term representation for the u component of the velocity field may be written as

$$u = \beta_1(x)z - \frac{\beta_2(x)z^2}{2}, \tag{2}$$

and the w component may be obtained from the continuity equation.

To solve for the mass transport the u and w velocity components are substituted into equation (1) in conjunction with a transformation of coordinates from x, z to $\chi(x)$, $\eta = zg(x)$ (Chao [6]). A series solution to the resulting equation and boundary conditions has been obtained in terms of universal functions [6] and will not be repeated here. Of interest to us is the local mass flux at the wall, j_w, where

$$j_w = -D\left(\frac{\partial c}{\partial z}\right)_{z=0} = (c_w - c_\infty)Dg\left(\frac{-\partial\theta}{\partial\eta}\right)_{\eta=0} \tag{3}$$

with $(\partial\theta/\partial\eta)_{\eta=0}$ prescribed in [6].

For the cone rotating in a non-Newtonian power-law fluid we obtain the following relations for the functions $\beta_1(x)$ and $\beta_2(x)$ from the work of Mitschka and Ulbrecht [8, 9]:

$$\beta_1(x) = a\left[\frac{(\omega\sin\alpha)^3}{K/\rho}\right]^{1/(1+n)} x^{1/(1+n)} \tag{4}$$

$$\beta_2(x) = -b\left[\frac{(\omega\sin\alpha)^{5-n}}{(K/\rho)^2}\right]^{1/(1+n)} x^{(3-n)/(1+n)} \tag{5}$$

where a and b are tabulated in Table 1. The parameter b has been obtained from the basic equations in conjunction with the values of a tabulated in [8, 9].

The average mass flux over the conical surface extending from $x = 0$ to $x = L$ is given by

$$J_w = \frac{2\pi\int_0^L j_w x \sin\alpha \, dx}{\pi L^2 \sin\alpha}. \tag{6}$$

Table 1

n	a	b
0·2	0·52821	−2·05924
0·4	0·50371	−1·37894
0·5	0·50052	−1·24146
0·6	0·50021	−1·15316
0·8	0·50378	−1·05149
1·0	0·51021	−1·00000
1·1	0·51383	−0·98428
1·3	0·52148	−0·96463
1·5	0·52917	−0·95483

Tabulation of parameters.

Substituting the relation for the local wall flux into equation (6) and carrying out the integration yields

$$\begin{aligned}\overline{Sh} = \frac{J_w L}{D(c_w - c_\infty)} = 4ASc^{1/3}Re_L^{(N_1+2)/4}\Bigg\{&\frac{1.11985}{N_1+4} - \frac{0.18868BSc^{-1/3}Re_L^{N_1/2}}{3N_1+4} - \frac{0.07271B^2Sc^{-2/3}Re_L^{N_1}}{5N_1+4} \\ &- \frac{0.05079B^3Sc^{-1}Re_L^{3N_1/2}}{7N_1+4} - \frac{B(N_1/N)Sc^{-1/3}Re_L^{N_1/2}}{3N_1+4} \\ &\times [0.05751 - 0.09861(N_1/N) + 0.11358(N_1/N)^2 \\ &\quad - 0.12004(N_1/N)^3 + \ldots] \\ &- \frac{B^2(N_1/N)Sc^{-2/3}Re_L^{N_1}}{5N_1+4} \\ &\times [0.03600 - 0.09161(N_1/N) + 0.10676(N_1/N)^2 \\ &\quad - 0.10557(N_1/N)^3 + \ldots] - \ldots\Bigg\}.\end{aligned} \tag{7}$$

With the one term velocity profile, only the first term remains in the brackets and the result is then identical to that given in [3, 5]. The parameters in equation (7) are defined as follows:

$$Sc = \frac{K(\omega\sin\alpha)^{n-1}}{\rho D}, \qquad Re_L = \frac{\rho L^2(\omega\sin\alpha)^{2-n}}{K}$$

$$N = \frac{7+5n}{2(1+n)}, \qquad N_1 = \frac{2(1-n)}{3(1+n)}$$

$$A = \left(\frac{aN}{9}\right)^{1/3} \qquad B = -\left(\frac{b}{2a}\right)\left(\frac{9}{aN}\right)^{1/3}. \tag{8}$$

RESULTS AND DISCUSSION

Equation (7) is rather long and although it appears to be tedious to evaluate, the repetitive nature of the terms greatly simplifies the calculations. Furthermore, for all the cases studied, only the first four terms within the brackets make a significant contribution to the sum. Recall that the expression with only the first term in the brackets is identical to the result obtained using the one term velocity profile [3, 5].

We are particularly interested in comparing equation (7) with the one term relation and some typical curves are presented in Figs. 2–4. The one term result corresponds to the horizontal line denoted by $Sc = \infty$. As anticipated, the one term result is of limited utility for moderate and moderately large values of the Schmidt number and for these conditions the more accurate result, namely equation (7), is recommended.

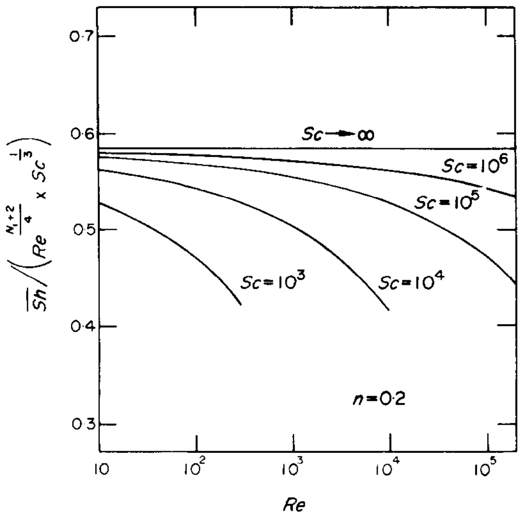

FIG. 2. Typical mass-transfer results.

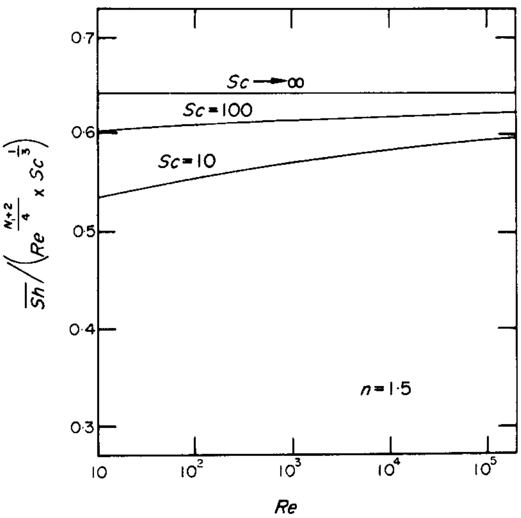

FIG. 3. Typical mass-transfer results.

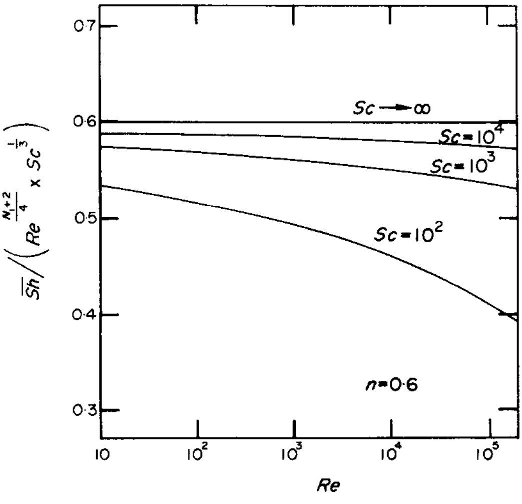

FIG. 4. Typical mass-transfer results.

The appropriate dimensionless parameters required to describe the mass transfer are n, Sc and Re_L, and are defined for a non-Newtonian power-law fluid in the previous section. For a Newtonian fluid, n equals unity and the Schmidt number is then a property of the fluid alone. However, when n is not equal to unity, the Schmidt number is a dynamic property of the system and results corresponding to a constant value of Sc do not correspond to a single fluid (with a given value of n). Indeed, each non-Newtonian fluid will have a range of Schmidt numbers associated with the range of rotational speeds for which measurements are made. This result makes the interpretation of data according to Figs. 2–4 rather complex.

We now consider the data of Greif *et al.* [2] for the rotating disk ($\alpha = 90°$) that was used to determine the diffusivity of oxygen in a non-Newtonian saline solution. For the fluids treated there, the Schmidt numbers were greater than 800 and the Reynolds numbers were less than 10^4. For this special range of parameters, the maximum error resulting from the one term Lighthill approximation [3] is found to be less than 5 per cent.* Hence, new values of the diffusivity based on the data of [2] and equation (7) will not be presented. It should be emphasized, however, that much larger errors can result for different experimental conditions (cf. Figs. 2–4) and then the more accurate relation for the mass flux, equation (7), should be used in determining the mass transport.

CONCLUSIONS

The laminar heat or mass transport to rotating cones or disks has been obtained for non-Newtonian power-law fluids. An exact expression has been obtained which is based on a two term representation of the velocity field. This result is an improvement to the one term Lighthill expression and provides quantitative limits for the range of validity of the one term analysis.

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*This is also true for the data obtained with the rotating cone [5].

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CONVECTION LAMINAIRE SUR DES CONES ET DES DISQUES ROTATIFS DANS DES FLUIDES NON-NEWTONIENS SUIVANT UNE LOI PUISSANCE

Résumé—On a évalué le transfert de masse en écoulement laminaire sur des cones et des disques rotatifs dans des fluides non-newtoniens suivant une loi puissance. Un résultat exact est présenté basé sur une décomposition en deux termes du champ des vitesses. Une comparaison avec les analyses précédentes basées sur un profil de vitesse à un seul terme définit les domaines de validité des résultats antérieurs.

LAMINARE KONVEKTION AN ROTIERENDEN KEGELN UND SCHEIBEN IN FLUIDEN, DIE NICHT DEM NEWTONSCHEN POTENZGESETZ FOLGEN

Zusammenfassung—Für laminare Strömung wurde die Stoffübertragung über rotierenden Kegeln und Scheiben in Fluiden, die nicht dem Newtonschen Potenzgesetz folgen, bestimmt. Ein genaues Ergebnis, das auf einer zweigliedrigen Darstellung des Geschwindigkeitsfeldes beruht, ist angegeben. Ein Vergleich mit früheren Analysen, die auf einem eingliedrigen Geschwindigkeitsprofil beruhten, definiert den Gültigkeitsbereich von früheren Ergebnissen.

ЛАМИНАРНАЯ КОНВЕКЦИЯ НА КОНУСАХ И ДИСКАХ, ВРАЩАЮЩИХСЯ В НЕНЬЮТОНОВСКИХ ЖИДКОСТЯХ СО СТЕПЕННЫМ ЗАКОНОМ

Аннотация — Исследуется массообмен при ламинарном течении около конусов и дисков, вращающихся в неньютоновской жидкости со степенным законом.

Представлен точный вывод на основе двухчленного представления поля скорости. Сравнение с более ранними исследованиями, основанными на одночленном представлении профиля скорости, устанавливает диапазон справедливости этих исследований.